## Title

Generation Capacity Expansion Planning under Hydro Uncertainty Using Stochastic Mixed Integer Programming and Scenario Reduction

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**Journal**: IEEE Transactions on Power Systems. Submitted: December 2013. Accepted: July 2014.

**URL**: `http://ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=6894248`

**DOI**: 10.1109/TPWRS.2014.2351374

## Abstract

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## Keywords

Generation expansion planning, mathematical programming, optimization methods, scenario reduction, stochastic mixed-integer programming, uncertainty

# Generation Capacity Expansion Planning under Hydro Uncertainty Using Stochastic Mixed Integer Programming and Scenario Reduction

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***Abstract*—Generation Capacity Expansion Planning (GCEP) is the process of deciding on a set of optimal new investments in generation capacity to adequately supply future loads, while satisfying technical and reliability constraints. This paper shows the application of Stochastic Mixed-Integer Programming (SMIP) to account for hydrological uncertainty in GCEP for the Chilean Central Interconnected System, using a two-stage SMIP multi-period model with investments and optimal power flow (OPF). The substantial computational challenges posed by GCEP imply compromising between the detail of the stochastic hydrological variables and the detail of the OPF. We selected a subset of hydrological scenarios to represent the historical hydro variability using moment-based scenario reduction techniques. The tradeoff between modeling accuracy and computational complexity was explored both regarding the simplification of the MIP problem and the differences in the variables of interest. Using a simplified OPF model we found the difference of using a subset of hydro scenarios to be small when compared with using a full representation of the stochastic variable. Overall, SMIP with scenario reduction provided optimal capacity expansion plans whose investment plus expected operational costs were between 1.3% and 1.9% cheaper than using a deterministic approach and proved to be more robust to hydro variability.**

***Index Terms*—Generation expansion planning, mathematical programming, optimization methods, scenario reduction, stochastic mixed-integer programming, uncertainty**

## NOMENCLATURE

*Indices and sets*

| | |
|---|---|
| $S$ | Index set of hydro scenarios. |
| $N$ | Index set of transmission buses. |
| $Y$ | Index set of years in the planning horizon. |
| $M$ | Index set of months (for hydro balance). |
| $T$ | Index set of Load Duration Curve (LDC) blocks. |
| $T_m \subset T$ | Index subset of LDC blocks in month $m \in M$. |
| $V$ | Index set of hydro storages. |
| $G$ | Index set of all generators. |
| $G^B \subset G$ | Index subset of generators with building decisions. |
| $G_v^H \subset G$ | Index subset of hydro generators associated to storage $v \in V$. |
| $G_i \subset G$ | Index subset of generators connected to bus $i \in N$. |
| $L$ | Index set of transmission lines. |
| $L_i \subset L$ | Index subset of lines connected to bus $i \in N$. |
| $L_{i-j} \subset L$ | Index subset of lines between buses $i, j \in N$. |
| $C^{NA}$ | Set of non-anticipativity (NA) constraints. |

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*Building variables (indexed yearly)*

| | |
|---|---|
| $Build_{y,g,s}^{Gen}$ | Integer decision to build units of $g \in G^B$. |
| $Units_{y,g,s}^{Gen}$ | Existing units (integer) of generator $g \in G^B$. |
| $Vio_{y,c}^{NA}$ | Violation of NA constraint $c \in C_{NA}$. |

*Dispatch variables (indexed by LDC-block)*

| | |
|---|---|
| $P_{t,g,s}^{Gen}$ | Generation of generator $g \in G$ (MW). |
| $USE_{t,n,s}^{Bus}$ | Unserved energy at bus $n \in N$ (MWh). |
| $P_{t,l,s}^{Line}$ | Power flow on line $l \in L$ (MW). |
| $\delta_{t,n,s}^{Bus}$ | Phase angle difference between bus $n \in N$ and the reference bus (rad.). |

*Hydro balance variables (indexed montly)*

| | |
|---|---|
| $W_{m,v,s}^{Vol}$ | Water volume of storage $v \in V$ (m$^3$). |
| $W_{m,v,s}^{Rel}$ | Water release of storage $v \in V$ (m$^3$). |
| $W_{m,v,s}^{Spill}$ | Water spillage of storage $v \in V$ (m$^3$). |

*Parameters*

| | |
|---|---|
| $\omega_s$ | Weight (probability) of hydro scenario $s \in S$. |
| $h_t$ | Hours in LDC block $t \in T$ (h). |
| $BC_{y,g}^{Gen}$ | Cost of building a unit of generator $g \in G^B$ ($). |
| $OC_{t,g}^{Gen}$ | Variable operation cost of generator $g \in G$ ($/MWh). |
| $Load_{t,i}^{Bus}$ | Net load requirement for bus $i \in N$ (MWh). |
| $VoLL_t$ | Value of lost load ($/MWh). |
| $Pen_{y,c}^{NA}$ | Penalty cost for violation of non-anticipativity constraint $c \in C^{NA}$ ($). |
| $Y_l$ | Susceptance of line $l$ ($1/\Omega$). |
| $\eta_g$ | Efficiency of hydro generator $g \in G$ (MWh/m$^3$). |
| $W_{m,v,s}^{Inflow}$ | Natural water inflow of storage $v \in V$ in month $m \in M$ and scenario $s \in S$ (m$^3$). This is the stochastic parameter. |

Depending on the type of variable, it will be indexed at the LDC-block, monthly or yearly level. All variables are also indexed per scenario. If $x$ is a variable, then $\underline{x}$ and $\widehat{x}$ denote its lower and upper bound, respectively. Some other symbols with a narrower scope are defined where they are used.

## I. INTRODUCTION

CAPACITY expansion planning (CEP) involves decisions about the investments to make in order to minimize the total operation and investment costs of a power system over a certain time horizon. CEP was first formulated as an optimization problem in 1957 [1], but it was only after development of computing and decomposition techniques that capacity expansion plans for simple models of real power systems [2], [3] could be obtained. Due to rapid changes in the industry since the 80's (faster load growth, demand management, fuel price uncertainty, and deregulation of the electricity sector) planning models have become even more important. In general, CEP problems can be formulated as a cost minimization or market surplus maximization problem in which operation costs are modeled through an optimal power flow (OPF) whose underlying structure can change because of some associated generation and/or transmission investment decisions. CEP problems generally consider separately the generation and transmission investment decisions, not only because of the infrastructure involved in each type of decision is different, but also because of the way each type of investment modifies the structure of the power flow formulation. While generation investments only modify the supply curve locally at each node (local modification of OPF), transmission investments modify transmission capacities and the admittance and impedance matrices of the associated power flow problems (global modification of OPF).

Particularly, the generation capacity expansion planning (GCEP) problem consists of deciding the location (where), timing (when) and sizing (how much) of new generation investments in order to minimize total system investment and operation costs, assuming that the transmission infrastructure investments are provided exogenously. The deterministic GCEP problem (assuming perfect foresight) has been widely studied and several methods to solve it have been proposed, either by using traditional linear programming (LP), mixed integer programming (MIP) [4]–[6], or heuristic/meta-heuristic techniques [7], [8]. Of particular interest lately has been the inclusion of reliability constraints [9], renewable integration [10], [11], emissions control [12], and decentralized decision making with incomplete information [13], [14]. Recent reviews of capacity expansion methods [6], [15] show that only a handful of papers deal with uncertainty in CEP, and most of them only do so either using heuristics or with *a posteriori* sensitivity analysis.

Two-stage stochastic mixed-integer programming (SMIP) has recently been proposed as a viable alternative to address the challenges posed by optimal power systems planning [16] under uncertainty. In [16] the authors also stressed the need for exploring the trade-off between modeling accuracy and complexity, as the optimization problem can easily become computationally intractable without sensible modeling simplifications. For example, [17] and [18] propose two-stage SMIP-based models for GCEP under demand and fuel price uncertainty, and in order to reduce the computational burden they use scenario construction and reduction schemes. In general, previous articles about using SMIP in CEP have been limited to theoretical discussions of the issues and have only been implemented in relatively small test systems (e.g. [2], [3], [16], [18]).

This paper presents a SMIP formulation for the GCEP problem under hydro uncertainty and shows its application to the Chilean Central Interconnected System (*Sistema Interconectado Central*, SIC), providing insights about how problem size (and scenario size in particular) affects simulation time. As the application to a real-sized system entails significant computational challenges requiring simplifications of the full problem, we discuss how scenario reduction affects the solution of the GCEP. To do this, this paper proposes novel metrics to evaluate the goodness of the reduced representation of the stochastic variable both in terms of the objective function value and in terms of the investment decisions.

The SIC is an hydrothermal system with about 45% hydroelectric generation capacity. Depending on hydrological conditions and the management of the water in the reservoirs the contribution from hydro energy can vary significantly from year to year (e.g. 41% to 70% of the total energy in 2012 and 2006, respectively). Thus, hydro uncertainty must be considered in both planning and operational models. While capacity expansion plans must take into account hydro uncertainty to ensure system adequacy during dry years, investors worry about not over-investing so that they receive an adequate level of return during wet years. However, incorporation of uncertainty in CEP usually implies the solution of very large optimization problems that can usually be solved only after making considerable simplifications. This is the case not only for hydro uncertainty, but also for demand growth and future fuel price uncertainty, among other types of uncertainties lurking in the shadows of any long-term modeling effort.

In order to adequately represent the statistical properties of the hydro variable in the complete set of historical scenarios (henceforth CSoHSc) we propose a methodology to select a subset of historical scenarios (henceforth SSoHSc) and their respective weights for its use in the SMIP formulation. The idea is that by selecting an adequate SSoHSc and adjusting the weights, the SSoHSc will have similar statistical properties to the CSoHSc but keeping the associated SMIP problem smaller.

There are three main objectives for this paper: 1) to propose a model based on SMIP for GCEP under hydro uncertainty, 2) to explore the trade-off between computational complexity and accuracy brought by hydro scenario reduction, and 3) to illustrate the use of SMIP to obtain generation capacity expansion plans for the SIC.

Section II of this paper presents the GCEP model and the SMIP formulation. Section III discusses the hydro scenario reduction. Section IV describes the case study and discusses simulation results. This section also proposes metrics to: 1) evaluate how well the CSoHSc is approximated by the SSoHSc from the point of view of the objective function, and 2) evaluate how the investment plans are affected as a result of using the SSoHSc. Finally, section V provides conclusions and directions for further research.

## II. SMIP APPLIED TO GCEP

GCEP lends itself nicely to SMIP, where investments are first-stage integer decisions (and are the same for all different realizations of the uncertain parameters) and operational decisions (second-stage decisions) are made after uncertainty realizes [19]. SMIP represents the probability distribution of the random parameters (in this case the hydro inflows) by a finite set of discrete scenarios [20]. In the proposed formulation, the first-stage variables (investment decisions of generation capacity) are the decisions to make under uncertainty and are unique for all the hydro scenarios. Although conditioned by the investment decisions, operational decisions such as generation dispatch and water usage (second-stage variables) are not subject to uncertainty as they are computed independently for each scenario.

The use of scenarios to represent uncertainty allows to formulate the SMIP starting from a deterministic MIP formulation as the minimization/maximization of the expected value of the deterministic objective function, which, using scenario-wise decomposition, turns out to be the weighted sum of the objective function for $S$ possible realizations of the uncertain parameter. The second-stage variables for each scenario must satisfy the full constraint set for the OPF problem.

Although using unit commitment (UC) models (instead of OPF) has been suggested for CEP (e.g. [16]), the UC formulation can bring additional complexity to an already difficult MIP problem for two reasons. First, it implies solving chronologically (instead of using LDC curve blocks), which tends to increase the number of second-stage variables. Second, UC decisions are integer variables which increases exponentially the size of the *Branch & Bound* tree.

Mathematically, SMIP is a particular case of MIP, which can be solved using a conventional MIP solver. Although conceptually the idea of applying stochastic programming to CEP is more than 20 years old (see for instance [2], [3]), only recently MIP progress and advances in computer performance are making the application of this technique to real systems possible.

### A. Objective function

In CEP, the objective function formulation depends on the goals of the planner. From an investor point of view, the objective function will be to maximize the profit, while from a central planner point of view the objective function will be the minimization of the net present value (NPV) of the investment plus operational costs or the maximization of the NPV of the social benefits. Although the second point of view does not necessarily replicate investments in a deregulated market, its results can be used to provide guidance to market participants and regulators and/or to suggest what to build and what to pay in capacity auctions [16].

The problem is formulated chronologically at monthly steps and non-chronologically within each month by decomposing the monthly load duration curve (LDC) in blocks. The GCEP objective function is as follows:

$$\begin{aligned}
&\text{minimize}\\
&\sum_{s\in S}\omega_s\left[\sum_{y\in Y}\sum_{g\in G_B}BC^{Gen}_{y,g}\cdot Build^{Gen}_{y,g,s}\right.\\
&+\sum_{t\in T}\sum_{n\in N}VoLL_t\cdot USE^{Bus}_{t,n,s}\\
&\left.+\sum_{t\in T}\sum_{g\in G}OC^{Gen}_{t,g}\cdot h_t\cdot P^{Gen}_{t,g,s}\right]\\
&+\sum_{y\in Y}\sum_{c\in C^{NA}}Pen^{NA}_{y,c}\cdot Vio^{NA}_{y,c}
\end{aligned}\tag{1}$$

The integer first-stage decision variables are $Build^{Gen}_{y,g,s}$ (investment decisions). Generators are built at the beginning of each year. Investment costs $BC^{Gen}_{y,g}$ are annualized using the weighted average cost of capital (WACC) specific for each project, which can incorporate the risk associated to each project and a reasonable return for each investor. Also, due to the length of the planning horizon, all coefficients in the objective function are discounted using the interest rate in order to obtain the net present value (NPV) of the total cost of each decision. The last year is repeated to perpetuity in order to minimize end-effects.

Second-stage variables are any operational decisions, such as generation, unserved energy, power flows, hydro flows, and storage volumes. Although some second-stage variables do not appear explicitly in (1), they appear in the set of constraints. The cost coefficients are indexed differently depending on how ofter they change.

### B. Constraints

GCEP has a large number of different constraints that must be respected for each scenario, among them energy balance at each node (2), flow definition and maximum capacity for each transmission line (3), generation limits (4), and hydraulic network constraints (5).

$$\begin{aligned}
&\sum_{g\in G_i}h_t\cdot P^{Gen}_{t,g,s}+USE^{Bus}_{t,i,s}+\sum_{l\in L_i}h_t\cdot P^{Line}_{t,l,s}=Load^{Bus}_{t,i}\\
&\qquad\qquad\forall t\in T,\forall i\in N,\forall s\in S
\end{aligned}\tag{2}$$

$$\begin{aligned}
&P^{Line}_{t,l,s}=Y_l\cdot\delta^{Bus}_{t,i,s}-Y_l\cdot\delta^{Bus}_{t,j,s},\quad l\in L_{i-j}\\
&\underline{P}^{Line}_{t,l}\le P^{Line}_{t,l,s}\le\widehat{P}^{Line}_{t,l}\\
&\underline{\delta}\le\delta^{Bus}_{t,i,s}\le\widehat{\delta}\\
&\forall t\in T,\forall i\in N,\forall l\in L,\forall s\in S
\end{aligned}\tag{3}$$

$$\begin{aligned}
&Units^{Gen}_{y,g,s}-Units^{Gen}_{y-1,g,s}=Build^{Gen}_{y,g,s}\\
&0\le Build^{Gen}_{y,g,s}\le\widehat{Build}^{Gen}_{y,g}\\
&0\le Units^{Gen}_{y,g,s}\le\widehat{Units}_g\\
&0\le P^{Gen}_{t,g,s}\le\widehat{P}^{Gen}_g\cdot Units^{Gen}_{y,g,s}\\
&\forall y\in Y,\forall t\in T,\forall g\in G,\forall s\in S
\end{aligned}\tag{4}$$

$$\begin{aligned}
&\sum_{t\in T_m}\sum_{g\in G^H_v}P^{Gen}_{t,g,s}/\eta_g+W^{Spill}_{m,v,s}-W^{Rel}_{m,v,s}=0\\
&W^{Vol}_{m,v,s}-W^{Vol}_{m-1,v,s}+W^{Rel}_{m,v,s}=W^{Inflow}_{m,v,s}\\
&\underline{W}^{Vol}_v\le W^{Vol}_{m,v,s}\le\widehat{W}^{Vol}_v\\
&\forall m\in M,\forall v\in V,\forall s\in S
\end{aligned}\tag{5}$$

The energy balance for bus $i$ in (2) indicates that the sum of all generation and unserved energy in $i$, plus the flows on the transmission lines connected to the bus must be equal to the demand, for every time-block and scenario. The set of constraints (3) states the basic constraints of a DC power flow (flow definition in terms of the phase angles, and flow and phase angle limits). The set of constraints (4) deals with generation limits. The first constraint in (4) calculates (for each year) the number of existing generating units depending on the building decisions for that generator in that year, the second and third one bound the maximum number of units of a specific generator that can be built (in total and per year). The last constraint in (4) limits the generation based on the number of existing units (integer). For example, if $Units^{Gen}_{y,g,s} = 0$, its generation $P^{Gen}_{t,g,s}$ will be forced to be zero. If $Units^{Gen}_{y,g,s} > 0$, the generation $P^{Gen}_{t,g,s}$ will be limited to the number of existing units multiplied by the maximum capacity of each unit. Finally, the set of constraints (5) balances the incoming and outgoing water in each reservoir and updates the volume of water stored on a monthly basis. Natural water inflow $W^{Inflow}_{m,v,s}$ is the stochastic parameter, and will be different for each scenario.

Violations over and under are treated as different variables, and some of the constraints can be relaxed (at a cost or penalty). The constraints previously shown represent only a simplified subset of all the constraints involved in GCEP. There are many other constraints, such as cascading hydro constraints, min water flows due to alternative uses of water, specific generator limitations, N-1 security constraints, and variables and constraints associated to transmission losses. An important set of constraints specific to SMIP are the *non-anticipativity constraints* (6), forcing investment decision variables to be the same for every scenario.

$$Build^{Gen}_{y,g,1} = Build^{Gen}_{y,g,s}, \forall s \in S \tag{6}$$

### C. Implementation

Since the GCEP problem contains a set of integer decisions (how many units to build), it corresponds to a MIP and can be solved through a Branch and Bound algorithm (B&B). The energy market simulations were conducted using PLEXOS, a MIP-based electricity market simulation and optimization platform suitable for both operational and planning studies [21]. Once the mathematical problem is formulated, it is solved using the optimization solver Xpress [22].

## III. Representation of hydro uncertainty

Long-term decision making models require the definition of likely future scenarios for some of the input parameters. One possibility is to use an average or representative scenario, neglecting the variability and uncertainty of the stochastic input and considering only its central tendency and assuming perfect foresight. Another option is to select a set of likely scenarios to include a representation of variability and uncertainty for the stochastic input. On the one hand, while using a single scenario can produce solutions faster, the investment decisions made are not robust as the system might perform badly if certain scenarios for the stochastic input materialize (e.g. if demand grows much faster or if the primary energy resource is less abundant than expected). On the other hand, the investment decisions made when considering multiple scenarios will have a better performance for a larger portion of scenarios, but the solution of the related optimization problem is more difficult to obtain. For the GCEP, the performance and robustness of any given solution can be tested *ex-post* by fixing the investment decisions and repeatedly testing them using an operational model for the different realizations of the input parameters, as Section IV-C will show.

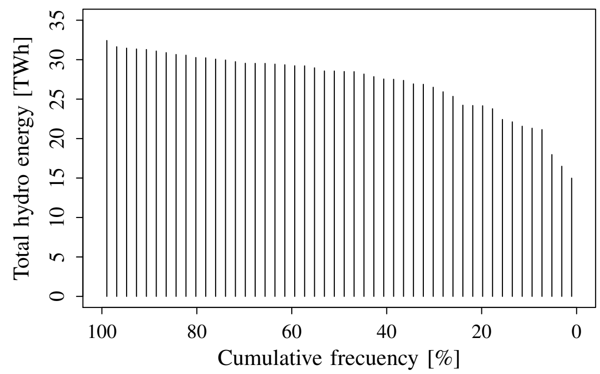


Fig. 1. Historical distribution of the (total) hydro energy per year in the SIC

### A. Hydrological data for the SIC

Historical hydro data is often used in hydro-thermal systems to represent hydrological variability (e.g. [23]–[26]). For the Chilean SIC, the use of historical hydro scenarios has been validated by independent consultants [23]. Hydro reservoir management and production cost modeling in the SIC usually considers 48 years of historic hydrological data [23], whose yearly energy historical distribution is depicted in Fig. 1. The full dataset consists of series of monthly averages of the water inflows for each run-of-river (RoR) generator and weekly averages for each hydro generator with storage capacity, from April-1960 to March-2009.

The water inflows dataset was converted (using the respective generators' efficiencies) to total equivalent monthly hydro energy, as these values are more directly related to the system operation costs of the GCEP problem. Then, the hydro energy was aggregated at the transmission bus level. Overall, each of the 48 hydrological scenarios consists of a vector with 1-year hydro data aggregated this way, containing 17 monthly time series and totaling 204 data points per scenario/year.

Each hydro scenario preserves the historical spatial correlation of the inflows and the monthly intra-year autocorrelation. Annual autocorrelation is not preserved, since the sequences in each year can be considered independent of those in the preceding years [27].

The use of the full set of hydrological scenarios may be suitable for hydrothermal coordination, but not for solving capacity expansion problems under uncertainty directly, because of the size and computational complexity of the mathematical problem [5], [28], [29]. Thus, in this work we have selected a reduced set of hydro scenarios which still considers the stochastic nature of the water inflows while keeping a manageable problem size. Thus, a SSoHSc will be used to represent the CSoHSc. Different approaches to the GCEP problem in terms of the representation of stochastic input parameters are illustrated in Fig. 2.

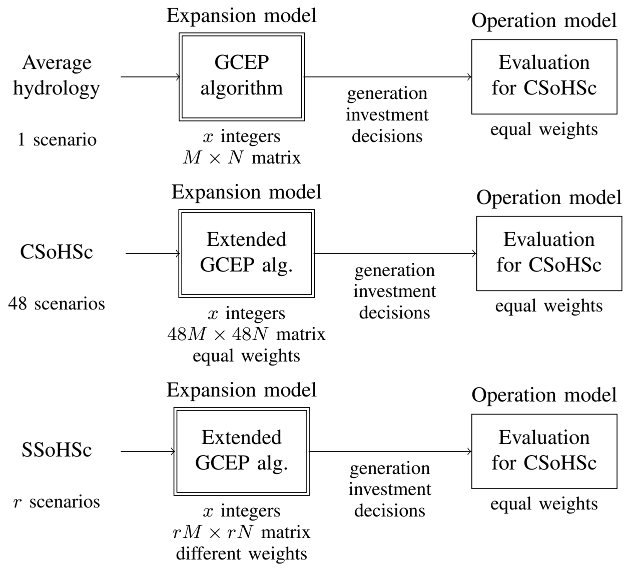


Fig. 2. Uncertainty representation approaches in the GCEP problem.

### *B. Scenario reduction methods*

One of the most commonly used options to perform scenario reduction when treating hydrological data is to select time series representing specific quantiles to represent the hydro variability. For example, the management model of the Highland Lakes by the Lower Colorado River Authority [26] in Central Texas uses a multi-stage scenario tree in which the nodes are selected as quantiles from the historical data. To represent the stochastic variable, weights for each scenario are calculated using the moment matching technique. A similar approach is taken in [30], where a linear formulation of the moment matching problem is used to adjust weights of scenarios previously generated by k-means.

Another set of techniques for scenario reduction in a more general context are the ones based on clustering [31], probability metrics [32], and importance sampling [33]. Those techniques are designed to select a large number of scenarios compared to which we can computationally afford in the GCEP problem and, since they are based on distances between scenarios, to perform selection in low-dimensional spaces. These are the two main reasons that led us to use an almost pure moment-matching scheme to select the SSoHSc.

### *C. Hydro scenario reduction using the moment-matching method*

The main assumption of the moment-matching method is that the CSoHSc can be approximated by a SSoHSc as long as the vector of moments of order $i$-th of the CSoHSc ($\boldsymbol{\mu}^{(i)}$) is similar to the vector of moments of the SSoHSc ($\boldsymbol{m}^{(i)}$). Thus, the moment-matching method tries to minimize the difference between the moments of the CSoHSc and the moments of the SSoHSC, which is achieved in two ways: (a) By selecting the subset of scenarios $S_r \subset S, |S_r| = r$ that minimize the error, and (b) by choosing a convenient weight $w_s$ for each scenario $s \in S_r$, as the moments depend on $w_s$. The subset $S_r$ determines the right-hand-side of (5), while the weights $w_s$ go in the objective function (1).

The optimization considers up to the fourth moment to take into account the negative skewness and the presence of extreme values in the full collection. Thus, the process minimizes differences in average, variance, skewness and kurtosis between the CSoHSc and the SSoHSc. Furthermore, if the hydro energy variable is conceived as a vector in which each component is the total energy inflow in a month of the year, then the covariance matrix represents the autocorrelations of the process. Additionally, the covariance matrix contains information about both the auto and cross-correlations between the energy injections at different points in the system, so minimizing differences in the covariance matrices should be considered in the optimization process. Thus, the problem of selecting the optimal subset of scenarios can be summarized as the minimization problem in (7)

$$\begin{aligned}\min_{S_r,\boldsymbol{w}} \quad & \left\{ \sum_{i=1}^{4} \gamma_i \left\| \boldsymbol{\mu}^{(i)} - \boldsymbol{m}^{(i)}(\boldsymbol{w}) \right\|_2^2 \right. \\ & \left. + \hat{\gamma}_2 \sum_{(k,p)\in K|p<k} \left(\mu_{kp} - m_{kp}(\boldsymbol{w})\right)^2 \right\} \qquad (7) \\ \text{s.t.} \quad & \boldsymbol{1}^T \boldsymbol{w} = 1, \quad \boldsymbol{w} = (w_{s_1}, \ldots, w_{s_r}) \\ & w_{\min} \le w_s \le 1, \quad \forall s \in S_r \subset S\end{aligned}$$

where $\mu_{kp}$ and $m_{kp}$ are the terms of the covariance matrix of the CSoHSc and the SSoHSc, respectively, and the $\gamma$ factors are used to rescale the difference of the moments into the same range and to properly weight each difference (differences on the first moment are much more important than differences in the fourth moment). Thus, the second term in the objective function corresponds to the difference in the bivariate moments (analogous to the elements in covariance matrix, but centered on zero instead of the respective averages). The weights of the objective function act as penalization parameters causing the differences of the moments multiplied by the weights to be on the same order of magnitude. Problem (7) is a non-linear combinatorial optimization problem. A detailed discussion of the calculation of the $\gamma$ factors and the algorithm for solving (7) can be found in [27]. Using (7), we selected 14 optimal SSoHSc, ranging from one to 14 elements in each one.

The solutions of the model described by (7) (the elements of the SSoHSc and their weights $w_s$) can be seen as inputs for the optimization problem described by Eqs. (1) to (6). In other words, (1) can be thought as an input-output model, where the inputs are the inflows and the output is the expansion plan. Therefore, if we replace the original probability distribution of the inflows by an approximated one with similar statistical properties, the output of the model described in Eqs. (1) to (6) should be similar. Formally, in problem (7) we are minimizing the difference between two probability metrics, using the difference in the statistical moments as a pseudo-distance function between them (this is not a distance function since zero difference in a finite number of moments do not imply that the distributions are identical). A similar argument is used in [32], derived from the notion of stability for stochastic continuous programs, to establish the minimization of a distance function between probability distributions as the criterion for scenario reduction. It is interesting to highlight that the moment-matching method is not limited to hydro data, as we have applied the same methodology to other types of multidimensional variables appearing in different power system applications, such as LDC decomposition and representation of variable renewable generation.

## IV. GCEP SIMULATION RESULTS

### A. Chilean Central Interconnected System (SIC)

The test system is the Chilean SIC. The SIC is a hydrothermal system with about 13.6 GW installed generation capacity of which approximately 44.6% is hydro. It has several baseload coal units (21.6% of total generation in 2012), some newer combined-cycle and open-cycle gas-fired units (15.2%), some fuel-oil and diesel-based peaking plants (18.4%) and hydro generation (41.1%). Two models of the SIC were built for the purposes of this work:

- A reduced model with an optimization horizon of 15 years, 71 investment options (new generators or expansions of existing ones) and 16 transmission buses. Transmission was modeled as a transportation network since there are not closed loops in the power flow (i.e. using the flow or the angle difference are equivalent).
- A full model with an optimization horizon of 15 years, 71 investment options, 78 transmission buses, and 5 LDC blocks per month. Transmission was modeled as a DC-OPF with linearized losses.

The MIP gap limit was set to 0.1% for both models. The reduced model is used to test the performance of the scenario reduction when solving the stochastic GCEP problem. The reduced detail in this model allowed solving the problem for the CSoHSc and comparing that solution with the solution using each of the SSoHSc, in order to evaluate the tradeoff between hydro uncertainty modeling accuracy and computational complexity. Afterwards, the stochastic GCEP is solved for the full SIC model using one of the SSoHSc to obtain a capacity expansion plan with a more adequate level of detail in the OPF while still being robust to the hydro uncertainty.

### B. Performance of scenario reduction in the GCEP problem

There are two main aspects of interest when measuring the performance of a scenario reduction scheme: one is the simplification in the MIP problem that reduction led to, and the other the difference in the variables of interest when solving the reduced MIP problem. This difference can also be seen as an approximation error between the solutions of the complete and the reduced MIP problems brought by the reduced representation of the stochastic hydro variable.

Regarding the simplification in the MIP problem, Table I shows the characteristics of the reduced optimization problems. Although the problem size increases linearly with the number of scenarios contained in the SSoHSc, as the integer variables (investment decisions) are first stage variables in the stochastic optimization, all MIP problems have the same number of integer variables (408). Therefore, the branch and bound (B&B) algorithm faces a tree of the same size when solving the different problems, but at each node the B&B algorithm has to solve LP problems of different size.

TABLE I
PROBLEM STATISTICS FOR THE REDUCED MODEL

| Hydro scenarios (Subset size) | Problem size (millions) Rows | Columns | Non-zeros | Runtime [s] |
|---|---|---|---|---|
| 1 | 0.16 | 0.24 | 0.52 | 54 |
| 2 | 0.32 | 0.48 | 1.03 | 308 |
| 3 | 0.48 | 0.72 | 1.55 | 143 |
| 4 | 0.64 | 0.97 | 2.07 | 978 |
| 5 | 0.79 | 1.21 | 2.59 | 1628 |
| 6 | 0.95 | 1.45 | 3.11 | 2309 |
| 7 | 1.11 | 1.69 | 3.62 | 1374 |
| 8 | 1.27 | 1.93 | 4.14 | 1516 |
| 9 | 1.43 | 2.17 | 4.66 | 4187 |
| 10 | 1.59 | 2.42 | 5.18 | 5488 |
| 11 | 1.75 | 2.66 | 5.70 | 3206 |
| 12 | 1.91 | 2.90 | 6.22 | 8581 |
| 13 | 2.07 | 3.14 | 6.73 | 11018 |
| 14 | 2.23 | 3.38 | 7.25 | 5866 |
| 48 | 7.64 | 11.60 | 24.87 | 145607 |

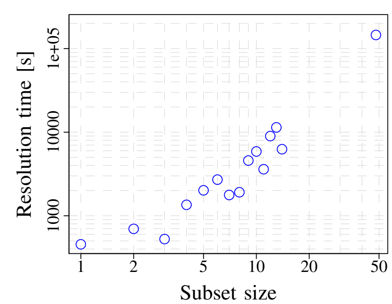


Fig. 3. Resolution time of the corresponding MIP problem for different sizes of the SSoHSc.

The optimization problem was formulated and solved using the software described in Section II-C. Simulations ran on a workstation with 2 Xeon E5-2630 processors and 32 GB of RAM. Running the stochastic GCEP with the full 48 hydro samples for the reduced model took over 40 hours of simulation time. Fig. 3 shows how runtime grows with the number of samples. The empirical relation between these two quantities is almost cubic, which is a consequence of the larger LP problems at each node of the B&B algorithm. For the same reason, the memory requirements grow along with the simulation time. Running the stochastic GCEP with all the 48 samples in the reduced model took the full 32 GB of RAM of the workstation. Therefore, scenario reduction is needed in order to obtain a solution for the stochastic problem in the full model with an adequate MIP gap.

Regarding the approximation error on the variables of interest, the differences in the solution of the GCEP problem using the SSoHSc instead of the CSoHSc can be measured in different ways. In this paper we show two L1-norm-based distance metrics to evaluate the differences in the variables of interest. The first metric is based on calculating the difference between expected and estimated total costs, that is, it evaluates the impact of the scenario reduction on the objective function. The second metric directly compares the capacity expansion plans, that is, it compares the investment variables.

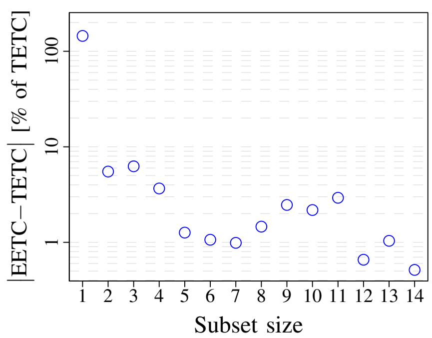


Fig. 4. Differences in the objective function between the solutions of the complete and the reduced MIP

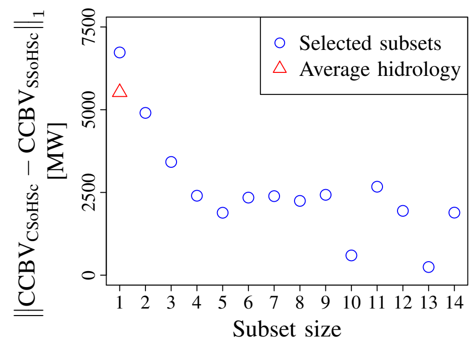


Fig. 5. Differences in the Cumulative Capacity Built Vectors (CCBV) between the solutions of the complete and the reduced MIP

For each capacity expansion plan, the first metric requires the calculation of the expected total costs as follows:

- Estimated Expected Total Costs (EETC): The objective function of the GCEP problem using a SSoHSc.
- True Expected Total Costs (TETC): The total cost of the expansion plan decided with a SSoHSc but whose true expected operational costs are post-calculated for the CSoHSc.

The differences between EETC and TETC for the capacity expansion plans obtained using different SSoHSc ($|\text{EETC} - \text{TETC}|$) are shown in Fig. 4. Notice that the capacity expansion plan (and therefore investment costs) for the calculation of both the EETC and the TETC is the same for a given SSoHSc. This metric assesses how well the operation costs (costs associated to second stage variables) are approximated by the use of a SSoHSc instead of the CSoHSc when solving the GCEP. As one would expect, such difference tends to decrease as the number of elements in the SSoHSc increases.

The second metric directly compares the differences introduced by the use of a SSoHSc in the first-stage variables with the solution obtained using the CSoHSc. This comparison is made using the Cumulative Capacity Built Vector (CCBV) of each investment plan, a vector containing the cumulative capacity built along the optimization horizon for each one of the 71 individual expansion options. We found the Mean Square Error (MSE) to be an inadequate metric in this context because it weights very heavily some relatively minor differences such as delays in the building decisions, distorting the measurement. For this reason we used a L1-norm-based distance metric of the differences between the vectors. Thus, the distance (as measured by the L1-norm) between two of these vectors ($\|\text{CCVB}_{\text{CSoHSc}} - \text{CCVB}_{\text{SSoHSc}}\|_1$) takes into account differences in location, sizing, and timing of the investments. The result of comparing the CCBV obtained for each SSoHSc with the one obtained with the CSoHSc is presented in Fig. 5. In general, as Fig. 4 and Fig. 5 show, the differences in the investment plans tend to decrease as the number of elements in the SSoHSc increases. Note that this decrease is not monotone as a consequence of system topology: while the scenario reduction is carried out considering the hydro energy injected at each node, the same energy put in two different locations of the system lead to different effects on the stochastic GCEP objective function costs. However the differences due to using a SSoHSc of two or more elements are always smaller than the error of using a single hydro scenario, justifying the use of SMIP instead of deterministic MIP for GCEP.

Although in this section we have used two metrics to evaluate the differences in the optimal solution caused by the scenario reduction, it is important to point out that they measure different things and estimating them pursue different objectives. The first metric ($|\text{EETC} - \text{TETC}|$, see Fig. 4) is measuring the approximation error in the objective function (1) due to the scenario reduction. This difference is used to define stability of stochastic programs as well as in the definition of the probability metrics used for scenario reduction [32], as it measures how well the probability distribution is approximated from the point of view of the objective function. The second metric ($\|\text{CCVB}_{\text{CSoHSc}} - \text{CCVB}_{\text{SSoHSc}}\|_1$, see Fig. 5) evaluates the difference in the solution for the first-stage variables, corresponding to the difference between investment plans due to the scenario reduction.

## C. *Performance of the SMIP solution*

Preliminary testing on the reduced model enabled us to examine both the performance of the scenario reduction and of using SMIP for the GCEP problem. The performance of stochastic programming can be measured by solving standard variations of the stochastic problem [20], whose objective function values are shown in Table II for the CSoHSc. EV is the objective function value of the deterministic GCEP problem using an average hydro scenario, and EEV is the expected cost of that investment plan when tested for all hydro scenarios. RP is the objective function value of the SMIP optimal solution. As expected, RP $<$ EEV because the RP solution considers the variability of the hydro inflows when obtaining the optimal investment plan. The *Wait-and-See solution* (WS) deterministically obtains different investment plan for each of the 48 hydro scenarios (assuming perfect foresight), so WS $<$ RP.

The differences between the values in Table II provide performance metrics for the SMIP solution, and are shown in Table III. The *Expected Value of Perfect Information* (EVPI), representing the expected value of having a perfect knowledge about the future is 353.8 MMUS$. The most important performance metric is the *Value of the Stochastic Solution* (VSS).

TABLE II
OPTIMAL OBJECTIVE VALUE OF TEST PROBLEMS

| Problem | Obj. Value [MMUS$] |
|---|---|
| EV (*Expected Value* solution) | 24016.9 |
| WS (*Wait-and-See* solution) | 25802.0 |
| RP (*Recourse Problem* solution) | 26155.7 |
| EEV (Expected result of EV) | 26665.4 |

TABLE III
STOCHASTIC SOLUTION PERFORMANCE METRICS

| Performance indicator | Value [MMUS$] | Value [% of EEV] |
|---|---|---|
| EEV - EV | 2648.5 | 9.9 |
| EEV - WS | 863.5 | 3.2 |
| VSS = EEV - RP | 509.7 | 1.9 |
| EVPI = RP - WS | 353.8 | |

The VSS is relatively large when compared to the objective function values, and covers almost 60% of the gap between the EEV and the WS model. These estimated savings are an important incentive for using SMIP in capacity expansion planning of hydrothermal power systems with significant hydro energy variability.

Naturally, investment plans obtained using the CSoHSc will perform better (in terms of costs) than a plan obtained using a SSoHSc. Fig. 6 compares the performance of the different plans with the *Wait-and-See* model ("48-samples IND", with capacity expansion plans obtained separately for each hydro scenario). Despite SMIP solutions not making significantly larger investments than the EEV solution ("01-mean"), they all show better global performance, as Fig. 7 shows.

Besides the expected economic benefits of the SMIP method when compared to a deterministic approach, the solutions obtained using SMIP showed to be more robust to hydro variability. To estimate the robustness of a specific investment plan, we calculated the yearly operation costs that the system would incur if each hydro scenario realizes. Then, the standard deviation (st. dev.) of these costs can be seen as a measure of

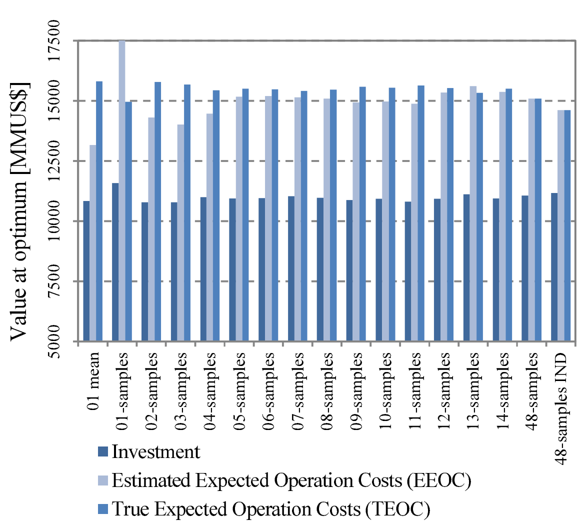


Fig. 6. Total costs for GCEP in the reduced model

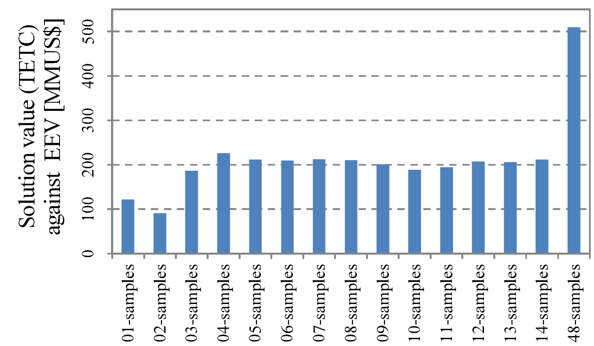


Fig. 7. Improvement (expected savings) of the stochastic solution against the EEV

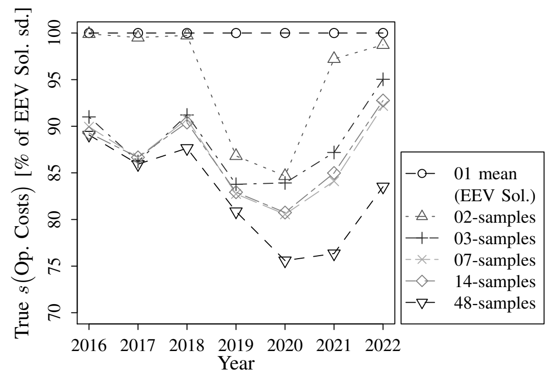


Fig. 8. Relative robustness of stochastic solutions with respect to the EEV

the variability in operation costs as a result of hydro variability. Fig. 8 shows this metric (normalized using the std. dev. of the yearly operation costs of the EEV) for a few selected capacity expansion plans. The std. dev. of the operation costs can be reduced up to a 15% in certain years by the use of a SSoHSc with only 3 elements (instead of the deterministic solution), while the reduction using the CSoHSc reach about 25%.

### *D. SMIP results on the full model*

The results for the reduced model shown in the previous sections were useful to compare the performance of the scenario reduction against the solution using the CSoHSc, as we found that running 48 hydro scenarios with a more detailed OPF model was not possible in a reasonable simulation time with the computational resources available. The detail in the reduced OPF model, although comparable with CEP studies around the world, was deemed insufficient to provide meaningful insights for the SIC. Thus, a compromise had to be reached between the detail of the stochastic hydro variable representation and the detail of the OPF.

After testing the GCEP formulation for a more detailed OPF model (74 buses, DC-OPF with linearized losses) with subsets of hydro scenarios of different cardinality we were able to obtain results with a SSoHSc with 7 elements after 44 hours of simulation time. Figure 9 shows how the aggregated SIC capacity evolves over time for the optimal solution. The SMIP plan showed a 1.3% improvement with respect to a deterministic formulation using a single hydro scenario (VSS)

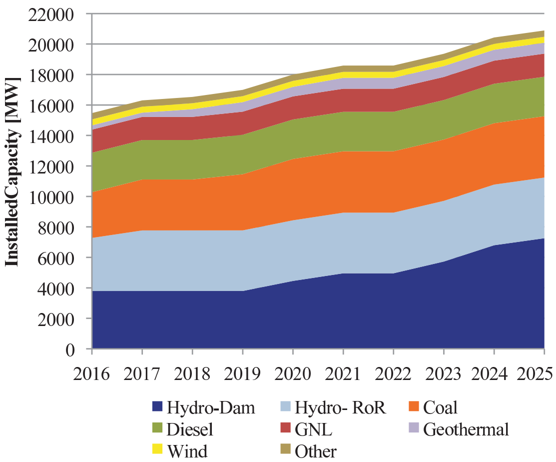


Fig. 9. SMIP capacity expansion plan for the full model

and proved to be more robust to hydrological variability when tested ex-post in a more detailed operational model.

## V. Conclusions

This paper presented a SMIP formulation for deciding future generation investments considering uncertainty on the hydrological resource. Due to the long decision horizons, optimal power systems planning presents a major challenge to researchers and practitioners because of the integer nature of investment decisions and the need to handle uncertainty about different parameters, e.g. primary resource availability, demand growth, fuel prices, emission prices, and technological innovation, among others. Although formulating a SMIP starting from a deterministic MIP is relatively straightforward, the formidable computational challenges of SMIP force to make aggregations and approximations to the full problem.

This paper provided insights on how problem size (and scenario size in particular) affects simulation time in SMIP-based GCEP. As scenario reduction is unavoidable in order to make the optimization problem tractable, this paper contributes to existing literature by proposing metrics to evaluate the goodness of the reduced representation of the stochastic variable in terms of the impact of such reduction on the solution of the associated GCEP problem. This impact was measured both in terms of changes in the objective function and changes in the investment decisions.

Ideally, the stochastic capacity expansion problems should be formulated using multi-stage stochastic optimization, as current decisions cannot anticipate what the future investment decisions will be. This paper does not address this aspect as the exponential growth in the number of scenarios can make the optimization problem intractable. Also, as suggested in [16], transmission and generation could be co-optimized using a similar SMIP formulation. However, we have found that including transmission investments decisions make the optimization problems considerably more challenging. In previous work, using a similar SIC database for co-optimizing generation and transmission investments under hydro uncertainty, we were incapable of using more than 3 hydro scenarios [5].

Despite all the challenges, the potential benefits of SMIP applied to optimal power systems planning can be quite significant, as the VSS metric showed in this paper. One of the most promising opportunities for drastically reducing simulation times is through decomposition using Benders' cuts or progressive hedging. This could lead to parallelizing the solution of the optimization problem, which has shown to be quite successful when dealing with the stochastic unit commitment problem [33]. This decomposition could be done either by time or by scenario, as it is done in stochastic unit commitment. Parallelization of the problem could also potentially allow to improve the model in different ways, such as increasing the transmission system detail, including additional scenarios, changing the OPF by a UC model, or co-optimizing generation and transmission investments.

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