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Energy Market Modelling

# Hydro Optimization in PLEXOS

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## 1 Executive Summary

Power systems having both hydro-electric and thermal generation require a systematic and coordinated approach in order to determine an optimal policy for dam operations. The goal of a hydro-thermal planning tool is to minimize the expected thermal costs along the simulation period. These types of problems generally require stochastic analysis to deal with inflow uncertainty. This can increase the mathematical size of the problem and can easily become cumbersome to solve.

PLEXOS® Integrated Energy Model offers many features to deal with the hydro-thermal coordination problem. In addition, it offers a seamless integration of phases, making it possible to determine an optimal planning solution in the medium-term and then use the obtained results in a detailed short-term unit commitment and economic dispatch (UCED) problem with increased granularity. For example, weekly targets as constraints filter down to produce hourly electricity spot prices. The following paper describes the most commonly used PLEXOS features for hydro optimization.

## 2 The Challenge

The systematic coordination of a system composed of both hydro-electric and thermal plants requires determining an operational strategy that for each stage of the planning horizon produces a scheduling plan of generation. This strategy minimizes the expected operational cost along the period, which is mainly composed of fuel costs plus penalties for failure in load supply. The problem becomes complex to solve because generally in hydro systems:

- Natural inflows (by nature) are stochastic processes.
- Availability of water stored in dams is limited.
- There are complex cascading hydro systems.
- Water usage policies and environmental releases such as irrigation settlements.

Water as a fuel supply is cost-free, but its opportunity cost is fundamental to finding the optimal strategy for operations. This issue creates the need for a decision in a given time period. Storages can't be drained too low, which might incur generation shortfalls or excessive thermal output. On the other hand, we also want to avoid spillage of water and lost generation opportunities. Figure 1 summarizes the dilemma a hydro power planner faces to operate a dam.

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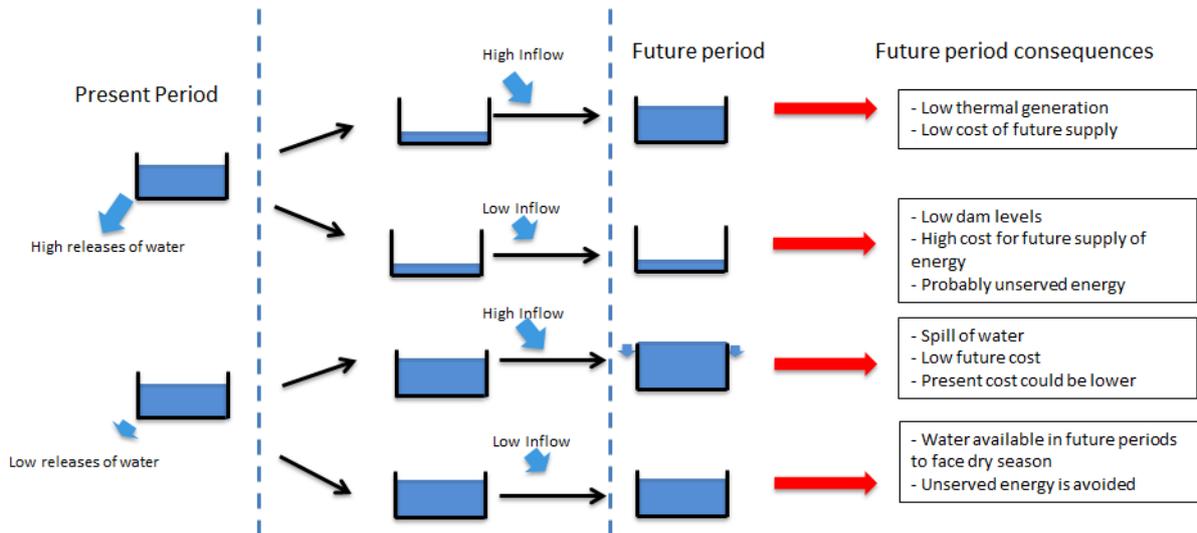


Figure 1: Diagram showing the dilemma hydro power planner faces under uncertainty

### 3 The Solution

PLEXOS® can find a hydro releasing policy that minimizes the expected thermal cost by formulating an optimization problem such as the following:

$$\text{Min } \{ \text{Variable Costs} \}$$

Subject to:

*Energy Balance Equation*

*FlowLimits*

*GenerationLimits*

$$\text{Hydro Balance: } S_{t+1} = S_t + Q_{\text{inflow}} - Q_{\text{release}}$$

The hydro balance equation shows the link between decisions in both the present and future.

Since water is free (no fuel cost) it is necessary to specify a final condition so as to minimise thermal costs along the simulation period, and to avoid the storage being completely drained. These final conditions can be represented as a target or a proxy for opportunity costs such as a deviation from targets, usually known as the future cost function or scrap value function.

To handle uncertainty in the inflows, PLEXOS offers stochastic optimization techniques in both 2-stage and multistage. Both techniques minimize the expected cost (or maximum benefits) of the system.

## 4 PLEXOS Features Explored

### 4.1 MT and ST schedule

To determine optimal targets or deviation targets, PLEXOS can analyse a longer horizon using Medium Term (MT) analysis and if the mathematical size of the problem is unsolvable a reduction in granularity (called demand blocks) can be applied to produce a mathematical problem that is faster to solve. Then the targets or deviation targets can be calculated from this optimal solution. This simplification produces an end volume at each block, then the MT problem passes the target or future cost function to each short term (ST) step so as to obtain a higher resolution release policy. Figure 2 shows an example of MT target (red line) and ST end volumes (blue area).

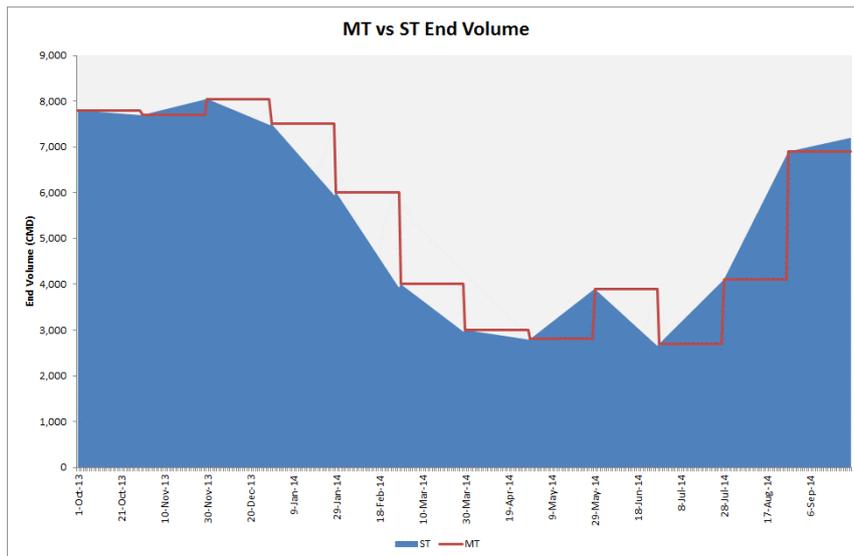


Figure 2: MT and ST with targets resolution

## 4.2 Cascading Hydro with Storage and Waterways

To create models of cascading hydro networks a combination of Storage, Waterway, and Generator objects can be used. PLEXOS offers the capability to model a cascading hydroelectric system like the one shown in Figure 3.

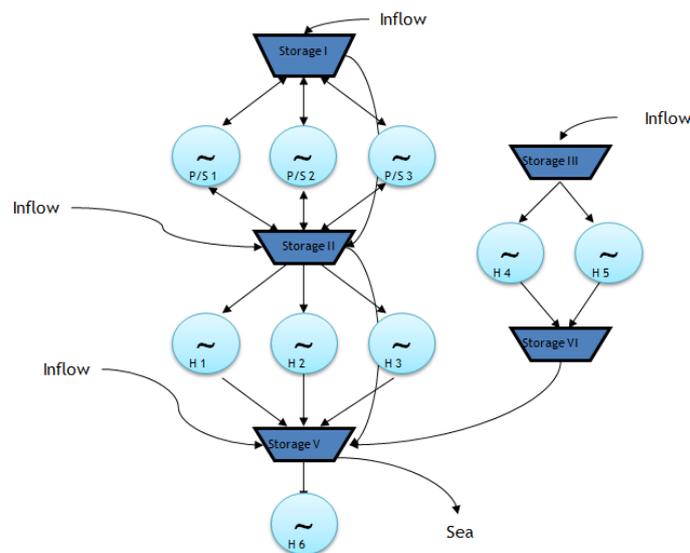


Figure 3: Schematic representation of Storages and Waterways

Storage objects are used to represent storage reservoirs with any storage capability or even simple junctions in a river-chain. Each storage can connect to one or more generators or waterways to create a model of a river chain.

Waterway objects connect the storages or spill water from the storage 'to the sea'. Waterway flows can delay the water flow, can have bounds placed in their flows and can limit the rate of change in their flows. The Constraint class makes it possible to relate water flow rates and generator efficiencies to storage elevations.

### 4.3 Pumped Storages Power Plants

Pumped storage plants store energy in the form of water. Pumps move water from a lower elevation reservoir (the 'tail' storage) to a higher elevation ('head' storage). To optimise the operation of a pumped storage power plant, PLEXOS formulates an optimization problem to decide when to release and pump water in order to minimize costs (or maximize benefits). In general one should expect that low-cost off-peak electric power is used to run the pumps and during periods of high electric demand and high price, the stored water is released through turbines to generate power.

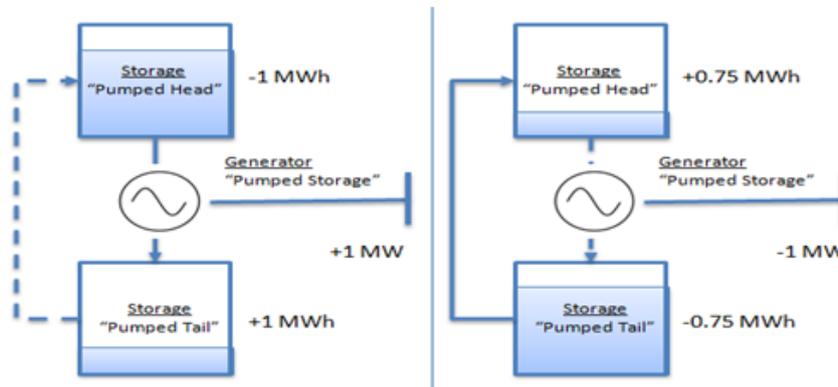


Figure 4: Pumped Storage cycle. Generation left hand side and pumping right hand side

### 4.4 Stochastic Optimization

PLEXOS can find the best outcome given the uncertainty in inflows. The problem is summarized in Figure 1 and can be solved using a stochastic optimization formulation. PLEXOS offers a two stages and multistage optimization approach. The difference is that multistage techniques allows a re-evaluation of the policy after some time, the period when this happens is called a "Stage". In a two-stage approach, the optimal policy is never re-evaluated because the information is not revealed at an intermediate stage during the simulation.

These two types of problems are represented in the below scenario trees.

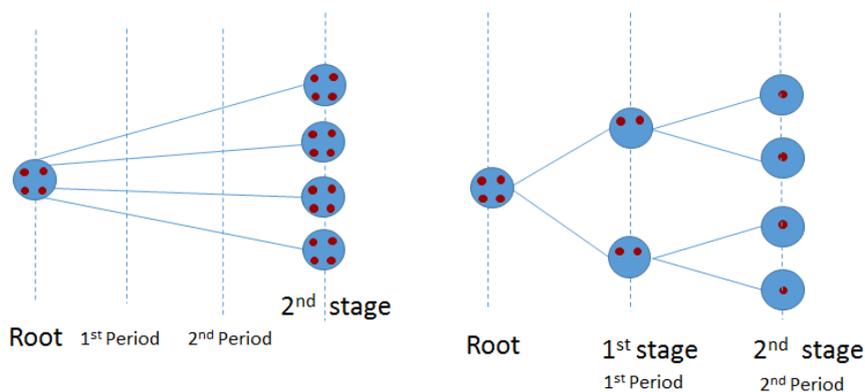


Figure 5: Left hand side represent 2 Stages SO, right hand side represents multi stage SO

## 4.5 Multi Stage Tree Reduction

When using multistage stochastic optimization, a large number of possible scenarios can be generated. For such a high number of scenarios, it is impossible to numerically obtain a solution for the multi-stage optimisation problem.

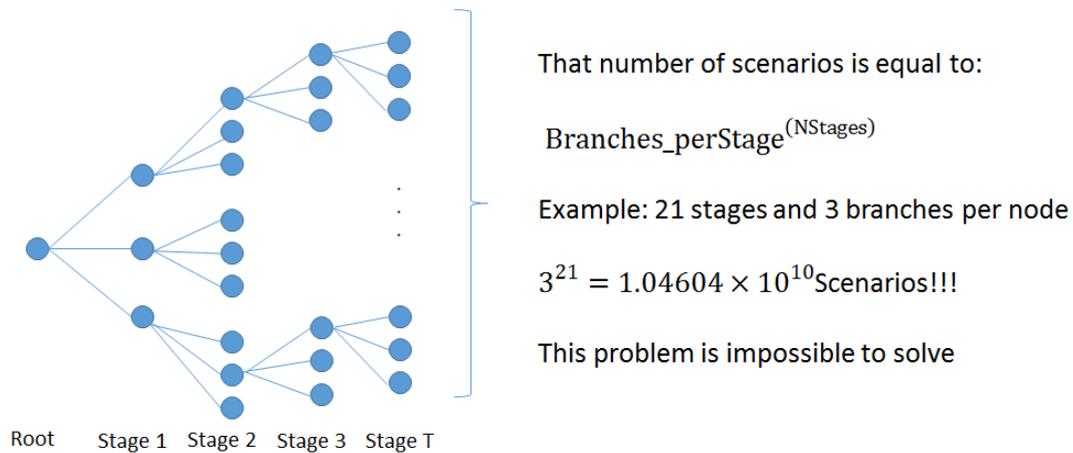


Figure 6: Multi Stage dimensionality issue

To help solve this problem, PLEXOS offers scenario reduction techniques. These techniques use strategies to reduce the number of scenarios in the optimization problem using algorithms for constructing a multi-stage scenario tree out of a given set of scenarios.

Since generating a very small number of scenarios by Monte Carlo Simulations is not desired because less scenarios give less information, the objective is to lose minimum information by the reduction process applied to the whole set of scenarios.

## 5 Case Study Results and Discussion

The case study summarized in Figure 7 illustrates the application of stochastic techniques in the hydro planning problem. The power system under study consists of 3 Power plants and there are 12 possible inflow sequences. The objective is to find an optimal trajectory for the storage along 1 year.

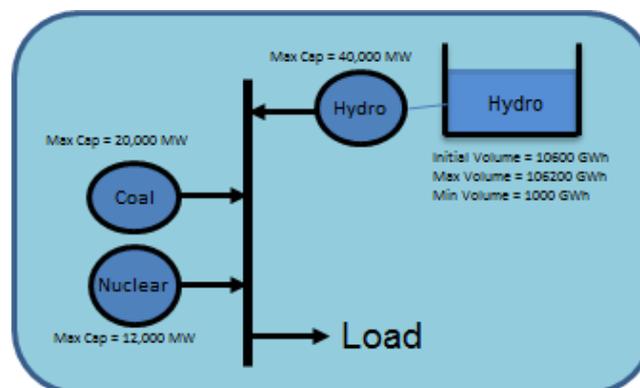


Figure 7: Left hand side schematic of the Power System under study, Right hand side load profile

Three simulations are performed: Deterministic, 2 Stage and Multistage. The optimal trajectories obtained are summarized in Figure 8.



Figure 8: Comparison of Trajectories for deterministic, 2 stages and multistage (4 stages).

2 stages obtains one single trajectory as there is no chance to re-evaluate the decision through time. On the other hand, multistage perform a re-evaluation at each stage, therefore producing a set of optimal past dependant trajectories. The deterministic approach gives optimal trajectories for each sample but it is not capable of answering the question “what decision do I have to make now given the uncertainty in the inputs?”

To evaluate the quality of the policies obtained, we can randomly select one of the policies, run the 12 inflow possibilities and calculate the expected costs. The results are summarised in Table 1.

Table 1: Summary of results

	Deterministic (Fixed Targets)	2 Stages	Multistage
<b>Expected Costs (Million \$)</b>	42,612	21,638	20,329

The results show that multistage stochastic optimization manages the storage better. Also 2 stages gives lower expected generation costs compared to the case with deterministic fixed trajectories.

## 6 Conclusions

Hydro–thermal power system modelling has lots of challenges and optimization techniques are necessary to provide the required answers. PLEXOS features can solve hydro–thermal coordination problems to ensure the user minimizes the cost or alternatively maximizes the benefits of a hydro–thermal portfolio. PLEXOS offers the flexibility to customise the stochastic resultant problem, providing the user with a powerful tool to manage hydro uncertainty that can be integrated with the rest of PLEXOS features.