

Modelling generator constraints for the self-scheduling problem

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Abstract: This paper deals with the self-scheduling problem faced by a powerplant when faced with a single price scenario. Powerplant's characteristics affecting its dispatching behaviour are described, such as maximum and minimum stable generation, minimum up and down times, time-dependent start-up costs and ramping rates. These characteristics are then translated into constraints applicable within a mixed-integer linear programming framework. Several formulations are presented which could be used in different modelling approaches. A case study involving a newly build gas-fired powerplant in Malženice is undertaken. The powerplant is dispatched against the hourly prices of the Slovak day-ahead short-term market (XMtrade®/ISOT) for 2011 delivery. Fuel is priced according to the official Slovak gas reference price and full pass-through of EU ETS carbon dioxide emissions price is assumed. Technical characteristics are based on publicly available information for a similar unit in Ireland. Several scenarios aimed at investigating the impact of individual constraints and horizon length were constructed. The results confirm the need to model detailed unit-commitment characteristics such as start-up costs using mixed-integer linear programming approaches since approaches based on Linear programming grossly overestimate solution value. The results also illustrate that selection of a simplified formulation may result in much higher computation speeds without sacrificing result accuracy. Finally, the results also demonstrate that the length of the horizon has a marginal impact on the solution quality, while having a significant influence on the computation speed.

Key words: Power generation dispatch, Power generation scheduling. Mixed-integer linear programming, Start-up costs.

Introduction

The self-scheduling problem (SSP) is a profit maximization problem faced by a generator, which sells power in a day-ahead market. The simple version of SSP is assumed a small (price-taking) generator along with its technological (and commercial) constraints and a single price forecast. The two simplifying assumptions both deal with the price: the single price forecast bears no uncertainty and the bids of a price-taker do not influence the resulting prices. Consequently, once the price uncertainties are removed, the problem is reduced to accurately implementing the various constraints faced by the generator. In such case, the optimal solution to the problem depends only on the technical characteristics of the generator. On the contrary in the real world forecasts are always uncertain implying a range of possible outcomes. Similarly, under specific conditions such as peak load, no generator is too small to have no market power. To deal with the above problems approaches such as robust programming (Landry, 2009) and oligopolistic models (Conejo, 2002), are necessary.

Similar but much larger problem is the unit commitment problem (UCP) faced by the Independent System Operator (ISO). This problem consists of selecting the generating units to be put into service, i.e. committed, during a scheduling period and deciding how much will each of them generate. The committed units must meet the system load and reserve requirements at minimum operating cost, subject to a variety of constraints. The individual generators have to meet their own technical constraints as well as the system-wide load and reserve constraints. In its most complex form, the UCP is highly non-linear due to quadratic losses within the network and non-linear relationships between fuel consumption and generation.

Historically a wide range of solution methods has been applied to both UCP and SSP. The methods include dynamic programming, Lagrangian relaxation, branch and bound, expert systems, fuzzy theorem, the hop field method, simulated annealing, tabu search, genetic algorithms, neural networks,

evolutionary programming and others. A bibliographical survey of various approaches is provided by (Padhy, 2004). The major limitations of the numerical techniques are the problem dimensions, large computational time and complexity in programming. Recently, due to advances in mixed-integer linear programming (MILP) solvers, formulations to both the UCP and SSP have been proposed (García-González, 2006).

The structure of the paper is as follows. The next section reviews the general technical characteristics of powerplants together with their causes. In the following section the SSP problem is formulated as MILP problem within which a powerplant's characteristics are captured as constraints. When several methods of formulating a constraint are possible, all are presented. The next section presents a case study for a small SSP and its results. The summary concludes and outlines areas of further research.

Powerplant technical and economic characteristics

Operating limits

Each generator can produce electricity within a certain range of values. The upper and lower limits - the *maximum capacity* P^{max} and the *minimum stable generation* P^{min} - typically refer to the limits of sustained generation under normal operating condition.

Production costs

The cost of production function is typically expressed as a quadratic function of the generation level p

$$c^p = \alpha + \beta p + \gamma p^2, \quad [1]$$

even though there is no fundamental reason for a quadratic relationship - it is merely a convenient approximation. In reality, the best a powerplant operator can do is to measure the steady-state fuel consumption corresponding to a certain generation level. Thereby the set of pairs $[p, c^p]$ is created, which is then approximated/extrapolated using a convenient method. Traditional UCP methods such as Lagrangian relaxation use a quadratic function or a higher order polynomial, while solution methods based on MILP use piecewise linear function(s). The production cost function need not be constant in time due to several reasons: the cost of the fuels is changing and the efficiency of the unit can be changing with the season (due to efficiency depending on the difference between the operating and the outside temperature).

Start-up and shutdown costs

As described in the section on operating limits, the unit can either produce within its operational range (unit is online) or have no output at all (unit is offline). The transitions between the offline and online states are known as the start-up and shutdown. During the start-up of a thermal powerplant additional costs are incurred. The start-up, which can last up to several hours, typically involves the heating up of the equipment in order to produce high pressure steam to drive the steam turbine. During this period fuel is consumed while no power is produced. In the case of some fuels such as lignite, the burning process has to be stabilised by fuel oil (lignite does not combust well at low temperatures) and thus additional costs are incurred.

In addition, during the start-up and shutdown, the unit is subjected to thermal stresses, which can lead to fatigue and possible permanent damage of the unit's components requiring future maintenance or repair. The sum of the average additional maintenance costs resulting from each start-up together with the total cost of the fuel used to bring the unit to within its operating limits is known as the start-up cost, C^u .

Contrary to the start-up process, when the change in temperature depends on the amount of heat supplied from the fuel and lost to the surroundings, during a shutdown heat exchange is due to losses only and follows an exponential decay function with the limit being the temperature of the surroundings, which is typically reached in the range of 48-60 hours. Therefore, the start-up cost depends exponentially (Christober, 2011) on the number of hours since the last shutdown, h^d_t ,

$$c_t^u = C^{u,c} \left(1 - \exp\left(-\frac{h_t^d}{H}\right) \right). \quad [2]$$

Cold start refers to a start-up of a powerplant that is at ambient temperature. The corresponding time it takes for the unit to cool down to this temperature level is known as the *cold start time*. Should the unit be restarted sooner, the amount of fuel needed to bring the unit to operational temperature would be lower. Depending on the time between the shutdown and the subsequent start-up, one can refer to a *hot start* (typically 6-8hrs and less) or a *warm start* (6-48hrs). These times and costs would normally refer to empirically determined values from typical start-up times: hot start – immediately after the unit is shut down, warm start – after the unit is shutdown over night, cold start – after the unit is shut down for the weekend.

The total start-up cost of a generator is the sum of fuel costs (depend on shutdown duration) and other start-up costs (typically a constant reflecting the increased maintenance)

$$C_t^u = FO_t^u FP_t + C^{u,other}. \quad [3]$$

Minimum up and down times

These constraints refer to the minimum time the unit has to be on once it starts up (MUT) and the minimum time it has to be off, once a shutdown occurs (MDT). MDT constraints arise due to necessary maintenance after a unit has been shut down, e.g. in the case of nuclear powerplants there is a technically mandated minimum down time of 15-24 hours (Schmid, 2010), while for flexible gas units no such a requirements exists. MUT constraints typically reflect the need to minimise thermal stresses in the equipment which could otherwise arise.

Ramp rates and start ramp rates

As has been previously suggested, for thermal units, rapid changes in temperature or output may lead to increased maintenance costs. Consequently, safe ramp up and ramp down rates are provided by the manufacturer. On occasions when the prices are high enough, the plant's operator may choose not to respect them, effectively exchanging the increased future maintenance cost for higher revenue in the short term. In addition, the physical design of a unit do provide an upper bound on these ramp rates, however these may be so high, that they need not be considered in the model. For example the ramp rate might allow for an increase from zero to full output in less then one model period. Such a constraint will never be binding.

Other constraints

Other constrains typically arise from commercial contracts or legislation. For example, the total generation within a horizon may be limited from both above and below due to constraints on the fuel. Commercial contracts may impose minimum and maximum daily or yearly offtakes (typically long term gas contracts). Similarly, emission caps may limit the total quantity of emissions (SOX/NOX/CO₂) generated (emissions are directly related to fuel consumption). Finally a broad category of constraints may apply to a particular problem. For example a generator may not be available on weekends or the transmission grid may limit the total generation of several units connected to the same node of the network.

Sources of revenue

Powerplant's main source of revenue typically comes from the sales of its generation. These can be realized on forward markets, day-ahead markets or markets with a very short lead horizon such as the intraday or balancing markets. In addition, the powerplant may receive additional payments from the TSO for providing network services. These may include ancillary services (services related to maintaining the balance of the system in real time) or capacity payments (payments designed to motivate the construction of otherwise non-profitable peak powerplants). Moreover, the powerplant may be eligible for payments from re-dispatching (situation when a powerplant is dispatched by the TSO not based on its economic schedule but in order to satisfy grid constraints). In this paper, for

simplicity reasons, only sales on the day-ahead market are be considered with the assumption that the full capacity is offered. On the other hand, the above-mentioned additional sources of revenue – if present – can change the economics of the powerplant considerably.

Problem formulation

Nomenclature

Indices

- i : generators, $i \in 1..I$
 t : periods in the modelling horizon, $t \in 1..T$
 k : production cost function segments, $k \in 1..K$

Decision variables (all lower indices i refer to generator i)

- p_{it} : Production quantity in period t [MW]
 l_{kit} : Production in the k -th segment of the pcw. linear approximation in period t [MW]
 c_{it}^p : Production cost in period t [EUR]
 c_{it}^u, c_{it}^d : Start-up and shutdown cost of generator i in period t [EUR]
 x_{it} : Unit commitment variable in period t (1 if online, 0 otherwise)
 y_{it} : Start-up variable in period t (1 if unit started in period t , 0 otherwise)
 $y_{it}^h, y_{it}^w, y_{it}^c$: Hot/warm/cold start-up auxiliary variable in period t
 $y_{it}^{h,exp}, y_{it}^{w,exp}$: Hot/warm start expiration binary variables
 z_{it} : Shutdown variable in period t (1 if unit shut off in period t , 0 otherwise)
 h_{it}^d : Number of hours generator has been offline in period t since the last shutdown [h]

Parameters (all lower indices i refer to generator i)

- α, β, γ : Constant/linear/quadratic coefficient of a quadratic production cost curve
 B_i : Linear coefficient of a linear production cost curve (constant efficiency)
 A_{0i}, B_{ki} : Constant/linear coefficient of a pcw. linear production cost curve with k segments
 $A_{ki}^{tan}, B_{ki}^{tan}$: Constant/linear coefficient of the k -th lower approximation of a production cost curve
 H_i : Characteristic time of a cooling function [h]
 λ_t : Market price in period t [EUR/MWh]
 P_i^{min}, P_i^{max} : Minimum and maximum generation capacity [MW]
 P_i^k : Maximum generation capacity of generator up to the k -th segment, $P_i^0 = P_i^{min}$, [MW]
 $C_{it}^{u,h}, C_{it}^{u,w}, C_{it}^{u,c}$: Hot/warm/cold start-up cost [EUR]
 $C_{it}^{u,other}$: Other start-up cost [EUR]
 FO_{it}^u : Fuel offtake at start-up in period t [GJ]
 FP_t : Fuel price in period t [EUR/GJ]
 $H_{it}^h, H_{it}^w, H_{it}^c$: Hot, warm and cold start-up time [h]
 $T_{it}^{min,u}, T_{it}^{min,d}$: Minimum up and down time [h]
 RR_{it}^u, RR_{it}^d : Ramp rate up and down [MW/h]

Simplest formulation of SSP

The objective function to be maximized is the generator's profit expressed as the difference between revenues and costs

$$\max \sum_{t=1}^T (\lambda_t p_t - c_t^p - c_t^u - c_t^d) \quad [4]$$

The basic characteristics of a generator is its maximum capacity, which sets an upper bound on generation in any instant

$$p_t \leq P^{max}, \forall t, \quad [5]$$

while the simplest formulation of production cost has constant unit costs

$$c_t^p = B p_t, \forall t. \quad [6]$$

Given a set of prices λ_t and cost of production characteristics (B) and assuming zero start-up and shutdown costs, c_t^u and c_t^d , the above equations define the whole SSP and can easily be solved using linear programming (LP) methods without the need for binary or integer variables. Provided that the linear cost function coefficient B approximates well the average production cost at maximum capacity, the above LP problem provides an upper bound on profit to all other formulations.

On the other hand, as has been suggested in the previous section, most real world units are much more complex. They cannot run at arbitrarily low output and there are limits on how quickly they can change their output and/or how quickly they can start. Apart from ramping rates, these additional constraints require the MILP framework. As is the case with any LP/MILP problem, additional constraints will result in a decrease of the objective function, i.e. profit.

The basic solution can be described purely in terms of powerplant's generation in each hour. The other quantities can be easily derived. The above formulation results in two possible solution values: either the powerplant generates at full capacity (in case its unit production costs exceed the market price) or the unit does not run (and the system load is served by other powerplants whose production costs allow them to be profitable at given power prices).

Unit commitment

The fact that the unit is either not producing electricity (unit off) or is producing in the admissible range (unit online) has to be modelled using a binary variable x_t :

$$\begin{aligned} x_t P^{min} &\leq p_t, \quad \forall t & [7] \\ p_t &\leq x_t P^{max}, \quad \forall t. & [8] \end{aligned}$$

The first equation implements the minimum stable generation, while the second one simultaneously implements the maximum generation limit (thus replacing equation [5]) as well as limits generation to 0, when the unit is offline.

Production costs

Equation [6] describes production cost as a linear function of the production quantity. Better approximation is obtained using a piecewise (pcw.) linear function

$$c_t^p = A_0 + \sum_{k=1}^K B_k l_{kt}, \quad \forall t, \quad [9]$$

where B_k are the unit production costs in the k -th segment of the pcw. linear approximation and l_{kt} are the corresponding generation variables. They are bound by the following constraints

$$l_{kt} \leq P^k - P^{k-1}, \quad \forall t, \quad \forall k. \quad [10]$$

The equation defining the total generation as a sum of generation at each segment is also required

$$p_t = P_0 x_t + \sum_{k=1}^K l_{kt}, \quad \forall t \quad [11]$$

Because the approximation is always greater or equal to the quadratic approximation, it is known as the *upper pcw. linear approximation*. One can also create a *lower pcw. linear approximation* by taking a set of tangents at points P_k (Figure 1b). This approximation does not divide the unit into several virtual smaller ones such as equation [9], but instead creates a lower bound on the production cost directly, using K equations

$$c_t^p \geq A_k x_t + B_k^{tan} p_t, \quad \forall t, \quad \forall k \in K. \quad [12]$$

The approximation improves with increasing number of segments k , however this also tends to increase the problem size. There are two approaches to this sub-problem: one tries to improve on the quality of the approximation without changing the model size, the other tries to iteratively refine the approximation in the neighbourhood of the solution.

(Wu,2011) deals with the former approach and finds the best segment partitioning method for the upper approximation. Traditionally, the segments $[P^{k-1}, P^k]$ are equidistant. Wu formulates the general

unconstrained optimization problem as a minimum of the sums of squared differences of the arc and chord lengths and solves it using Newton-Rhapon methods for optimal P^k .

(Viana, 2011) works with the lower approximation, starting with just two segments: one corresponding to the tangent at P^0 and the other to the tangent at P^k . After solving the problem with this simple approximation the solution p_t^* in period t is found. The method works by adding additional constraints of the type [12] to the problem and resolving. The constraints are only added to the periods when the unit is switched on. The method converges when the delta between the approximation of the cost function using existing tangent envelope and a new envelope created by adding a new constraint is less than a predefined ε .

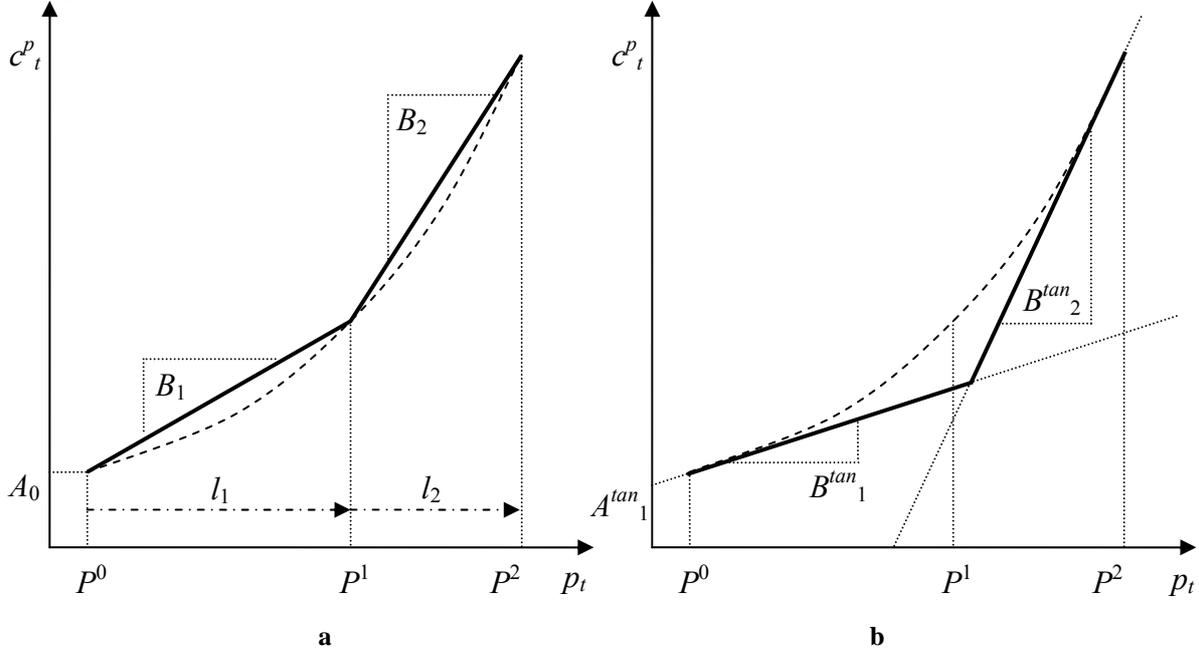


Fig. 1: Approximations (bold) of the quadratic production cost function (dashed):

a) Upper pcw. linear approximation using segments. b) Lower pcw. linear approximation using tangents.

Both the lower and upper pcw. approximations work only for convex (monotonically non-decreasing) functions, i.e. only if B_k form an increasing series. Non-convex functions require the use of additional binary variables b_k , which are non-zero only if the level of generation is within the corresponding segment k . The approximation is carried out using a pcw. linear method. The necessary equations are:

$$l_{kt} \leq (P^k - P^{k-1})b_{kb}, \quad \forall t, \quad [13]$$

$$l_{kt} \geq (P^k - P^{k-1})b_{k+1,b}, \quad \forall t, \quad [14]$$

$$b_{kt} \geq b_{k+1,b}, \quad \forall t. \quad [15]$$

Equation [15] forces an order in which the binary variables take on the value 1. Equations [13-14] replace equation [10], ensuring that the generation at the previous segment is at its maximum before the next segment is utilised.

Start-up and shutdown costs

Even though they are not strictly necessary, the model formulations are simple if the variables representing a powerplant's start-up and shutdown are explicitly defined. Their lower bounds are defined by how the unit commitment variable, x_t , changes in time:

$$x_t - x_{t-1} \leq y_t \leq 1 \quad \forall t, \quad [16]$$

$$x_{t-1} - x_t \leq z_t \leq 1, \quad \forall t. \quad [17]$$

These variables need not be defined as binary, but they effectively behave as such. The simplest start cost formulation (Landry, 2009) assumes that there are 3 start-up states (hot/warm/cold) and the start-up costs are a pcw. constant function of the time offline.

$$c_t^u \geq C^{u,h}(x_t - x_{t-1}), \quad \forall t, \quad [18]$$

$$c_t^u \geq C^{u,w} \left(x_t - \sum_j^{H_w} x_{t-j} \right), \forall t, \quad [19]$$

$$c_t^u \geq C^{u,c} \left(x_t - \sum_j^{H_c} x_{t-j} \right), \forall t. \quad [20]$$

The above formulation implies that the hot/warm/cold start-up costs apply as soon as the unit is down for one/ H_w / H_c periods. The formulation is easy to generalize to a higher number of start-up states. Conversely, only one of the equations is necessary to model a single start-up cost applicable after H_x periods.

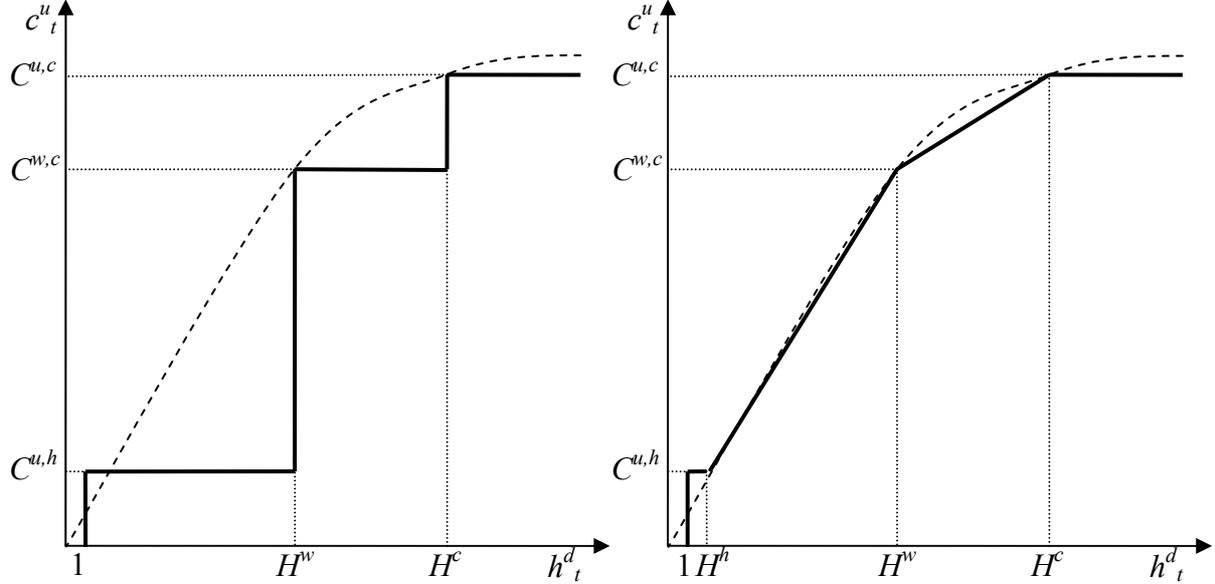


Fig. 2: Approximations (bold) of the exponential start-up cost curve (dashed):

a) piecewise constant, b) piecewise linear.

(Viana, 2011) works with two start-up states only, however allows for the start-up costs to vary linearly in between the two start-up times. Power market modelling software *PLEXOS for Power Systems* developed by (Energy Exemplar, 2011) generalizes this approach into multiple start-up states resulting in the following 10 equation formulation for 3 start-up states. Below, the formulation is broken down into 3 groups of equations.

The first group defines the start-ups and the corresponding costs:

$$y_t^h \leq 1, y_t^w \leq 1, y_t^c \leq 1, \forall t, \quad [21]$$

$$y_t = y_t^h + y_t^w + y_t^c, \forall t, \quad [22]$$

$$c_t^u \geq C^{u,h} y_t^h + C^{u,w} y_t^w + C^{u,c} y_t^c, \forall t. \quad [23]$$

The first equation sets the upper bounds on the auxiliary hot/warm/cold start-up variables. The second defines generator's start-up period t as being hot, warm, cold or their mix. The last equation defines start-up costs to be at least equal to an affine combination of the hot/warm/cold start-up costs.

$$h_t^d - h_{t-1}^d \geq 1 - Tx_b, \forall t \quad [24]$$

$$h_t^d \leq T(1 - x_t), \forall t \quad [25]$$

$$h_t^d - h_{t-1}^d \leq 1, \forall t \quad [26]$$

The next group of equations insure that that the counting variable h_t^d , which counts the number of hours that unit has been down, takes on the correct value in period t :

- if the unit is down in period t , the counter increases, (equation [24]),
- if the unit is on in period t , the counter is reset to zero (equation [25]),
- the maximum increase of the counter is 1 (equation [26]).

Finally, the last set of equations takes care of the linear interpolation. Two new binary variables $y^{h,exp}_t$ and $y^{w,exp}_t$, which reach value 1, when the linear interpolation between the hot/warm and warm/cold start-up times is no longer applicable, i.e. it expires:

$$y^h_t + y^{h,exp}_t \leq 1, \quad \forall t, \quad [27]$$

$$(H^w - H^h)y^h_t - (H^w - h^d_t) \leq (H^c + T)y^{h,exp}_t, \quad \forall t, \quad [28]$$

$$y^w_t + y^{w,exp}_t \leq 1, \quad \forall t, \quad [29]$$

$$(H^c - H^w)y^w_t - (H^c - h^d_t) \leq (H^c + T)y^{w,exp}_t, \quad \forall t. \quad [30]$$

Equation [27] determines the transition from a hot start to a warm start. As long as $h^d_t \leq H^h$, the start is hot. In case of the equality, i.e. $h^d_t = H^h$, y^h_t must take on value 1, so that the expiration variable on the RHS is not activated. When $H^h < h^d_t < H^w$, in order not to activate the expiration variable, the LHS must be at most zero. This is satisfied as long as

$$y^h_t \leq \frac{(H^w - h^d_t)}{(H^w - H^h)}, \quad [31]$$

i.e. y^h_t is inversely proportionate to the to the position of the current hours down h^d_t on the segment $[H^h, H^w]$. Finally, when $h^d_t \geq H^w$, there is no permissible positive value for y^h_t and the expiration variable $y^{h,exp}_t$ on the RHS is activated. Because this variable is binary, equation [26] insures that the hot start variable is indeed 0. The second pair of equations enforces a similar behaviour for the transition from warm to cold.

Minimum up and down times

Minimum up and down time constraints are very simple to formulate. The equation below shows the minimum up constraints for a minimum up time of $T^{min,u}$ periods

$$y_t \leq x_{t+k}, \quad \forall t, \quad \forall k \in 1.. T^{min,u} - 1. \quad [32]$$

The equation ensures that, if the unit starts in period t , it has to be online until the period $t + T^{min,u} - 1$. The formulation for minimum down time is similar:

$$z_t \leq 1 - x_{t+k}, \quad \forall t, \quad \forall k \in 1.. T^{min,d} - 1. \quad [33]$$

Ramp rates

Most thermal units have limits on how quickly they can ramp up and down in order to avoid thermal stresses leading to failure or increased maintenance costs:

$$p_t - p_{t-1} \leq RR^u, \quad \forall t, \quad [34]$$

$$p_{t-1} - p_t \leq RR^d, \quad \forall t. \quad [35]$$

This formulation works for units without a minimum stable generation P^{min} and also if the ramp rates are greater or equal to the minimum stable generation ($P^{min} \leq RR^u$ and $P^{min} \leq RR^d$). In the opposite case, the equations need to be modified to allow for the unit to start-up, e.g.

$$p_t - p_{t-1} \leq (RR_u + P^{min})x_t - P^{min}x_{t-1}. \quad [36]$$

Case study

Case study data

To illustrate the effects of modelling accuracy on the solution, a SSP problem has been set up, which tries to mimic a real world situation as close as possible. German utility EON has commissioned a new 430MW combined cycle gas turbine (CCGT) powerplant near the Slovak town of Malženice in January 2011 (EON, 2011). The SSP will try to model the optimal dispatch of the CCGT assuming a perfect forecast of the actual prices on the relevant market (Slovak day-ahead power exchange XMtrade®/ISOT). Since no information about the relevant gas price was available (commercial fuel contracts are often confidential), it was assumed that the relevant gas price is the official Slovak reference gas price (Slovakia, 2010), even though information in the press suggests EON's gas price could be lower due to contract renegotiations in 2010. In addition, the full pass-through of the opportunity cost of CO2 emissions under the EU ETS scheme was assumed. The emission allowance

prices were taken from (EEX,2011). The inclusion of emission costs does not modify any of the formulations presented above, it only changes the absolute value of fuel prices.

Given the lack of detailed technical data on the Malženice CCGT, data for a similar CCGT commissioned in June 2010 in Aghada, Ireland, was used. Aghada's technical characteristics are captured in the Irish Single Electricity Market modelling study (SEM, 2010) in a format suitable for power market modelling software *PLEXOS*. Since both units are very similar (both have a single-shaft configuration, both have similar P^{max} and both have similar reported efficiencies), the SEM data for the Aghada CCGT data will be used without additional modifications.

The SEM dataset contains the following characteristics for all powerplants:

- Operating range: P^{min} , P^{max} (winter, summer)
- Operating cost: No load cost (A_0), marginal heat rates (equivalent to B_i). Marginal efficiency is 65.6% and the corresponding full load efficiency is 57%
- Ramping rates: RR^u , RR^d
- Minimum up and down times
- Start-up: Fuel offtake at hot, warm, cold start and the corresponding start-up times

The data relevant for the Aghada CCGT is shown in Table 1.

Table 1: Aghada CCGT characteristics Source: SEM, 2010

Property	Value	Units	Property	Value	Units
P^{max}	431.6	MW	H^c	72	hrs
P^{min}	215	MW	$FO^{u,h}$	1200	GJ
P^0	215	MW	$FO^{u,w}$	1800	GJ
P^1	431.6	MW	$FO^{u,c}$	2400	GJ
A_0	354.511	GJ/hr	$T^{min,u}$	4	hrs
B_1	5.497	GJ/MWh	$T^{min,d}$	4	hrs
H^h	1	Hrs	RR^u	22	MW/min.
H^w	12	Hrs	RR^d	22	MW/min.

In addition, the assumption that the unit is initially out and has been so for more than H^c periods was made.

Case study formulation

This data was used within a market modelling tool Plexos with XPRESS-MP as the MILP solver. A base model was set up and additional 6 sensitivity models were run. The base model has all the constraints from equations [4], [7]-[11], [16], [21]-[30], [32] and [33], initialized using data from table 1. In addition, the base model uses a rolling horizon of 6 days, i.e. 144 periods, split into two groups (3+3 days). When the 6 day problem is solved only the solution of days 1-3 is saved and the model is then rerun for the next 6 day step, i.e. days 4-6. This set-up was chosen to accurately capture the longest constraint in the problem formulation, which is a cold start after 3 days. This way the solution of day 3 takes into account the most remote future constraint with direct impact on that day.

The following sensitivities were run:

- Horizon sensitivity 1: model with horizon of 2 days (1+1)
- Horizon sensitivity 2: model with horizon of 14 days (7+7)
- Production cost sensitivity: modelled using constant efficiency equal to P^{max} efficiency
- Start cost sensitivity: only warm start-up costs modelled
- Constraint sensitivity: No MUD, MDT constraints
- Simple SSP (LP): No MUD, MDT, P^{min} constraints, no start-up costs
- Fuel sensitivity: constant fuel price (equal to average 2011 price)

The goal of the first two sensitivities is to determine the impact of the modelling horizon, the next 3 test impact from simplifying and excluding some constraints and the final case tests the sensitivity of the base case to fuel price.

Case study results

All models finished under 3 minutes time on a Pentium M@1.86 GHz with 2 GB of RAM, except the model with 14 day (7+7) horizon, which took almost 20 minutes to finish and was 7 times slower than the base model. On the other hand, the model with a short 2 day (1+1) horizon finished in only 40% of the time. This suggests that solution time increases rapidly with longer horizons. Comparing the different horizons further, it is evident that both costs and revenues increase with longer horizons. The latter increase at slightly faster rate resulting in the objective function (Net profit) value being highest for the 7+7 horizon. However the overall change is less than 0.5%. The number of starts is almost constant.

The results of production cost sensitivity (Scenario Constant Efficiency) are not surprising as the only difference to the base model is higher efficiency when running close to P^{min} levels. Consequently, the model is able to sustain generating at these levels with lower costs resulting in decrease in number of start-ups as well as overall increases in generation, fuel and emission cost and most importantly in higher net profit.

Removing the MUT and MDT constraints (Scenario No MUT, MDT) from the formulation allows the powerplant to be more flexible and either capture price spikes in single periods or return online soon after shutdown. Consequently generation and net profits increase, even if only marginally.

Table 2: Results for Slovak reference price scenarios

Property	Base (3+3)	1+1	7+7	Const Efficiency	Single Start Costs	No MUT, MDT	Simple SSP (LP)	Const Fuel Price	Units
N. of Start-ups	167	168	168	140	159	167	121	149	-
Fuel Price	7.98	7.98	7.98	7.98	7.99	7.98	8.08	8.36	EUR/GJ
Fuel Cost	52021	50266	52332	58585	51377	52001	63099	46269	EUR000
Emission Cost	5295	5111	5329	5981	5211	5293	6194	4058	EUR000
Start-up Cost	2570	2556	2597	2226	2528	2570	0	2452	EUR000
Total Gen. Cost	59887	57933	60258	66791	59116	59864	69293	52779	EUR000
Price Received	65	66	65	64	65	65	65	68	EUR/MWh
Generation	1028	994	1034	1162	1014	1028	1236	872	GWh
Pool Revenue	67108	65178	67474	74518	66293	67086	80641	59054	EUR000
Net Profit	7222	7245	7216	7727	7177	7222	11349	6274	EUR000
Solution Time	164	68	1187	162	12	157	4	164	s

Allowing the powerplant to be subject to warm starts only (scenario Single Start Costs) decreases the costs of start-ups after long downtimes and increases them after short ones. Given the fact that the base solution has approximately 3 starts per week, it is likely to have both hot and cold starts. For this particular situation, this simplification results in slightly higher average start-up costs. On the other hand, simplifying the start-up cost function yields a significant 14-fold increase in computation speed. This sensitivity essentially replaces equations [21]-[30] with a single, much simpler equation [19].

The Simple SSP (LP) model which assumes no MUD, MDT and P^{min} constraints as well as no start-up costs (and can thus be solved using LP) illustrates the need to include such constraints in the problem formulation. This model's solution has 20% higher generation and 50% higher net profit.

The final sensitivity (scenario Const Fuel Price) illustrates that all of the above conclusions depend heavily on the fuel (and power market) price assumptions. A constant fuel price throughout the year decreases the number of starts and total generation by approximately 10%. The annual 3TWh generation suggested by the powerplant owner (EON, 2011) implies access to gas prices below the Slovak reference gas price.

Conclusion

This paper has described a powerplant's characteristics affecting its dispatching behaviour and translated them into constraints applicable within a MILP framework for solving the simple

scheduling problem of generator dispatch against a single price scenario. A case study involving a newly build gas-fired powerplant in Malženice has been undertaken. The results confirm the need to model detailed unit-commitment characteristics such as start-up costs. On the other hand they also illustrate the need for accurate modelling in terms of constraint formulation detail and appropriate horizon selection.

Future extension of the research could focus on improving the model formulations in terms of solution speed without sacrificing model accuracy as well as on the inclusion of additional characteristics required for more accurate modelling, such as the generation during a start-up or powerplant's participation in ancillary services market along with the associated revenues.

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Shrnutí

V tomto textu byly popsány charakteristiky elektrárny ovlivňující její dispečerské řízení. Tyto charakteristiky byly zachyceny jako omezení v problému smíšeného celočíselného programování, v němž je elektrárna provozována za účelem maximalizace zisku proti jedinému cenovému scénáři. Tento problém byl zachycen na konkrétní případové studii pro nově postavenou paroplynovou elektrárnu v Malženicích na Slovensku. Výsledky studie potvrzují nutnost zachycení technicko-ekonomických omezení, jako jsou například startovací náklady. Výsledky také ilustrují potřebu přesného zachycení těchto omezení spolu s vhodnou volbou modelovacího horizontu.

Budoucí rozšíření výzkumu se bude soustředit na zlepšení formulace modelu pro zrychlení výpočtu bez ztráty kvality a přesnosti řešení. Mezi další možné okruhy rozvoje patří také zachycení dalších charakteristik, jako je například výroba elektřiny při startu či zapojení elektrárny na trhu s podpůrnými službami.